\textbf{The MAIN TOPIC}

The existence of 2-\textit{factorizations} of a complete undirected graph on an odd number of vertices is always guaranteed and it seems clear that the number of non-isomorphic ones becomes huge as the number of vertices increases. Nevertheless, to enumerate them up to isomorphism is very hard and a general classification seems impossible to achieve without requiring some additional conditions. These conditions may involve the automorphism group of the 2-factorization as well as the types of 2-factors. Moreover, it seems reasonable that mostly 2-factorizations are rigid, i.e. with a trivial full automorphism group. This means the subclass of rigid 2-factorizations asymptotically covers the class of all 2-factorizations. In this context, the study of 2-factorizations with a non trivial automorphism group can lead to either partial or complete classification results. For example the classification of 2-factorizations which posses an automorphism group having a 2-transitive action on the vertex-set was achieved in [BIM]. In this last case the request on the group is strong and only few factorizations are permitted in such a particular situation.

Weaker conditions on the automorphism group are requested for the 2-factorizations presented here. Classification results are not achieved, but many properties are pointed out.

\textbf{Preliminary definitions and notations}

Given a positive integer \( r \) we denote by \( K_r \) the complete undirected graph on \( r \) vertices and by \( V(K_r) \) and \( E(K_r) \) its vertex-set and edge-set respectively.

\begin{itemize}
\item A 2-factor of \( K_r \) is a 2-\emph{spanning subgraph} and a 2-\emph{factor}, usually denoted by \( F \), is a collection of 2-factors whose edges partition the edge-set of \( K_r \).
\end{itemize}

The number of 2-factors in a 2-factorization is \( r - 1 \).

A 2-factor of \( K_r \) is a 2-\emph{factor} of \( K_r \) if and only if it is a 2-factor of \( K_r \).

The dihedral group of order \( 2n \) can be presented as follows:

\begin{align*}
D_{2n} = \langle g, x : g^n = x^2 = 1, gx = g^{-1} \rangle
\end{align*}

The following results can be found in [BR2] and [BR3].

\textbf{Proposition 1} If \( G \) is a 2-factorization, then \( G \) is a 2-factorization. \( G \) is a Hamilto-

\textbf{Some examples}

\begin{itemize}
\item \( A \)-factor of \( K_r \) which is 1- \textit{pyramidal} under \( \alpha \)
\end{itemize}

\textbf{Proposition 2} Any 1- \textit{pyramidal} 2-factorization of an undirected complete graph provides a solution to a suitable Oberwolfach problem and the 2-factors are in a unique orbit under the action of the group.

\textbf{Main results when \( k \geq 1 \)}

The following results can be found in [BR2] and [BR3].

\textbf{Proposition 3} There exists a 2-factorization under a dihedral group of order 2n if and only if 2n + 1 - \emph{at} \( t \) for some positive integer \( t \). Every such 2-factorization is of type \( 3, 4, \ldots, t \) and is generated by a 2-factor of the form

\begin{align*}
F = \{N(x_1, y), \ldots, N(x_s, y)\} \quad \text{where} \quad \{x_i, y\} = \{1, \ldots, s\}
\end{align*}

\textbf{Proposition 4} Let \( r = k_1 + k_2 + \ldots + k_t + 1 \) with \( k_i \geq 2 \) for each \( i \). A 1- \textit{pyramidal} solution for \( OP(2k_1, 2k_2, \ldots, 2k_t) \) under the action of the cycle group cannot exist in each of the following cases:

\begin{itemize}
\item \( n = 2 \) (mod 4);
\item \( n + r \) is an odd integer.
\end{itemize}

\textbf{Main results when \( k = 1 \)}

The following results can be found in [BIM].

\textbf{Proposition 5} Finite dihedral groups together with abelian and hamiltonian groups of order \( n \) realize 2-factorizations, where \( k = 1 \).

\textbf{Proposition 6} Each group \( G \) of order \( 2n \) realizes a \( 2n \)-\textit{pyramidal} 2-factorization.

\textbf{Proposition 7} Let \( G \) be a group of order \( 2n \). If \( G \) is a 2-factorization, then, for each odd \( t \), \( k \leq t \leq 2n - 1 \), \( G \) also realizes a 2-factorization.

Given a group \( G \) of even order, we denote by \( \lambda_G \) the minimum odd integer \( k \) for which \( G \) realizes a 2-factorization. Many examples of groups \( G \) with \( \lambda_G = 1 \) are presented in the side table (case \( k = 1 \)). These examples can be extended to realize \( k \)-\textit{pyramidal} 2-factorizations for each \( k \), \( 1 \leq k \leq |G| - 1 \).

\textbf{Main results when \( k = 1 \)}

\textbf{Proposition 8} Let \( G \) be an abelian group of order \( 2n \) and \( n \) \textit{in} \( G \). It is \( \lambda_G = 2l + 1 \).

\textbf{Proposition 9} Let \( G \) be a dihedral group of order \( 2n \). Then

\begin{align*}
\lambda_G = \left\lfloor \frac{n}{2} \right\rfloor + 2
\end{align*}

\textbf{Bibliography}

\begin{itemize}
\item [BR3] M. Buratti, G. Rinaldi: A non-existence result on cyclic k-decompositions of the cocktail party graph, submitted.
\end{itemize}